PRADIS

METHODS OF FORMATION AND NUMERICAL CALCULATION OF THE MATHEMATICAL MODELS OF THE TRANSIENT PROCESSES

PROGRAM SET FOR THE AUTOMATION OF THE SIMULATION OF NONSTATIONARY PROCESSES IN THE MECHANICAL SYSTEMS AND THE SYSTEMS OF OTHER PHYSICAL NATURE

VERSION 4.2

Content

1. Simple example

Let us examine oscillatory system with one degree of freedom. The body of mass ***m*** is connected with the fixed base by means of the elastic spring. Its motion occurs in the medium with liquid resistance under the action of external force (Fig. 1.1.). The position of body in the space and its initial velocity are assigned for the initial moment of time. It is necessary for each moment of time in the range from ***t0*** to ***t[konech]*** to determine displacement, speed and acceleration of body.

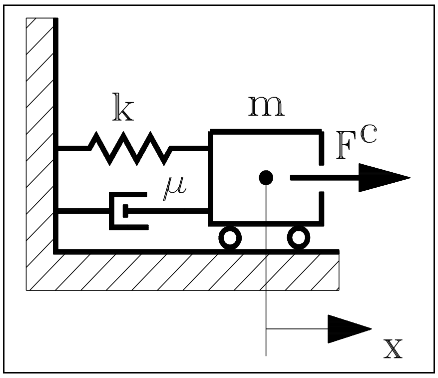


Fig. 1.1

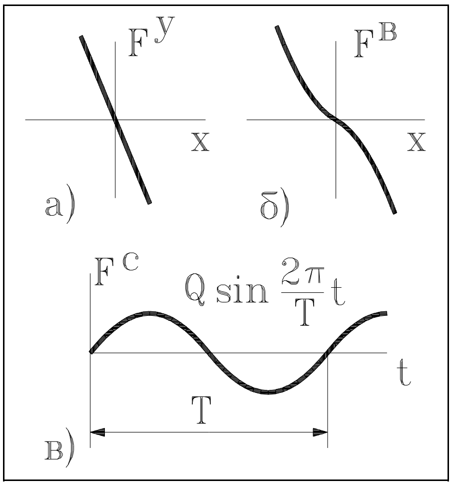


Fig. 1.2

Since it is necessary to illustrate solution of nonlinear problem, we will consider that the force of liquid resistance is proportional to the square of the relative speed of the ends of damper (Fig. of 1.2.[b]). The elastic force of spring linearly depends on displacement (Fig. of 1.2.[a]), the influencing force has the sinusoidal nature (Fig. of 1.2.[v]).

The dependences, which make it possible to obtain differential equation of motion, in accordance with second Newton's law take the form:

$$\boxed{ma = F^c + F^y + F^B} \qquad (1.1)$$

or

$$\boxed{F^{c}+F^{y}+F^{B}+F^{И}=0}, \quad (1.2)$$

where taking into account the selected directions ([ris].1.3):

$$\boxed{F^{c}=Q\sin\frac{2\pi}{T}t}$$

$$\boxed{F^{y}=-kx}$$

$$\boxed{F^{в}=-\mu\ v|v|} \qquad (1.3)$$

$$\boxed{F^{u}=-ma}$$

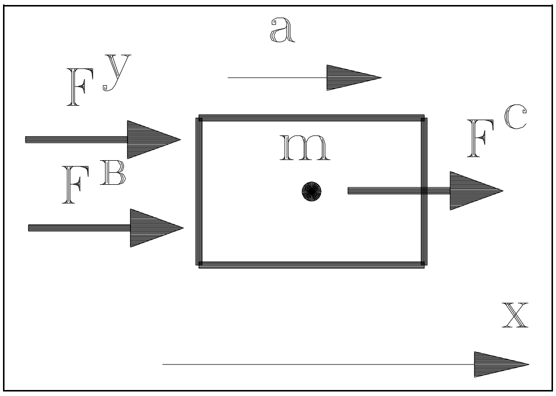


Fig. 1.3.

Here ***x***, ***v*** and ***a*** - respectively, displacement, speed and the acceleration of body.

The substitution of relationships (1.3) into equation (1.2) gives:

$$\boxed{-kx-\mu v|v|-ma+Q\sin\frac{2\pi}{T}t=0} \qquad (1.4)$$

Taking into account that

$$\boxed{v=\frac{dx}{dt}} \quad \text{and} \quad \boxed{a=\frac{d^2x}{dt^2}}, \quad (1.5)$$

we obtain the differential equation of motion of the body:

$$\boxed{kx+\mu\frac{dx}{dt}\left|\frac{dx}{dt}\right|+m\frac{d^2x}{dt^2}-Q\sin\frac{2\pi}{T}t=0} \qquad (1.6)$$

The use of a numerical approach to the integration of equation (1.6) assumes the presence of approximate solution for specific moments of time, i.e., temporary axis is represented by the totality of points ***t0, t1, t2,... ti, ti+1,... tn*** (Fig. 1.4), in each of which the approximate solution of equation searches for (1.6).

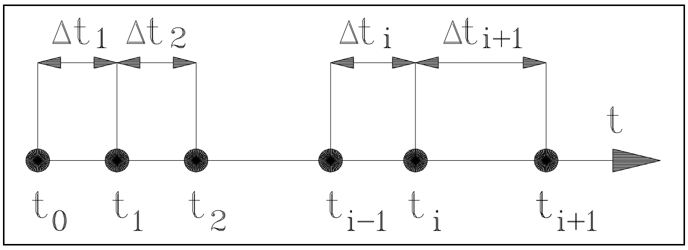


Fig. 1.4.

Integration is achieved consecutively, the selection of the value of the sequential step **Οφ δτ ι** depends both on the required indices of accuracy and on the results of integration for the already passed temporary points.

Thus, the use of a numerical approach to the solution of equation (1.6) makes it possible to pass from the continuous values ***x, v, a*** in entire time interval from ***t=0*** to ***t=t[konech]*** to the set of discrete values ***xi, vi, ai*** for specific moments of time ***ti***. In this case the algebraic formulas of the selected method of integration substitute differential relationships (1.5). Thus, the formulas of the implicit one-step method of Stormer establish the following dependence for the variables ***xi, vi*** in terms of th values ***of xi-1, vi-1*** [1] e known from the previous step:

$$x_i = x_{i-1} + v_{i-1}\Delta t_i + a_i \frac{\Delta t_i^2}{2} \qquad (1.7)$$

$$v_i = v_{i-1} + a_i \Delta t_i,$$

where $\Delta t_i = t_i - t_{i-1}$, $i = \overline{1, n}$

t0 - initial time,
x0, v0 - initial values of displacement and speed,
tn - finite time.

The values ***x0*** and ***v0*** must be known for the initial moment of time ***t0***. Setting aside for the moment a question of the selection of the value of the step of integration ***ti***, let us determine values ***x1***, ***v1, a1*** for moment of time ***t1 = t0 +D t1***.

Equation (1.4) for moment of time ***ti*** takes the form:

$$kx_i + \mu v_i |v_i| + mq - Q\sin\frac{2\pi}{T} t_i = 0 \qquad (1.8)$$

We supplement this relationship with the formulas of the selected method of integration (1.7) and we obtain for moment of time ***t1*** the closed system of equations:

$$kx_1 + \mu v_1 |v_1| + mq - Q\sin\frac{2\pi}{T} t_1 = 0$$

$$x_1 = x_0 + v_0 \Delta t_1 + a_1 \frac{\Delta t_1^2}{2} \qquad (1.9)$$

$$v_1 = v_0 + a_1 \Delta t_1$$

Let us reduce the obtained system to one equation, after expressing unknowns ***x1*** and ***a1*** through ***v1***:

$$a_1 = \frac{v_1 - v_0}{\Delta t_1} \qquad (1.10)$$

$$x_1 = x_0 + \frac{v_0 + v_1}{2} \Delta t_1$$

We obtain:

$$\boxed{k(x_0+\frac{v_0+v_1}{2}\Delta t_1)+\mu\, v_1|v_1|+m\frac{v_1-v_0}{\Delta t_1}-Q\sin\frac{2\pi}{T}t_1=0}$$ (1.11)

Grouping cofactors with the identical degrees of unknown $v1$, relationship (1.11) can be written down in the form:

$$\boxed{\alpha\, v_1|v_1|+\beta\, v_1+\gamma=0}$$, (1.12)

where $\boxed{\alpha\ =\ \mu}$

$$\boxed{\beta\ =\ \frac{k\Delta t_1}{2}+\frac{m}{\Delta t_1}}$$

$$\boxed{\gamma\ =\ kx_0+k\frac{v_0\Delta t_1}{2}-\frac{mv_0}{\Delta t_1}-Q\sin\frac{2\pi}{T}t_1}$$ (1.12[a])

Let us note that relationship (1.12) preserves its form for any moment of time ti during the appropriate replacement of subscripts (1 on i, 0 on i -1).

Thus, the use of formulas of the method of integration makes it possible to leave from the differential relationships on the time and converts initial differential equation (1.6) to the nonlinear equation of form (1.12), which must be solved at each step on the time.

Equation (1.12) is solved by Newton's method. Let us allow itself to resemble the sequence of actions during the solution of nonlinear equation by Newton's method.

The equation of the form is examined:

$$\boxed{f(z)=0}$$, (1.13)

where $f\ (z)$ - nonlinear function relative to unknown z.

The algorithm of numerical solution includes the following steps:

1) the selection of initial approximation to the solution - value $z0$;

2) the organization of the sequence of the iterations, for each of which is refined the obtained on the previous iteration value z according to diagram (Fig. 1.5.a):

$$\boxed{z^j=z^{j-1}+\Delta z^j}$$ (1.14)

$$\boxed{\Delta z^j=-\frac{f(z^{j-1})}{f'(z^{j-1})}}$$,

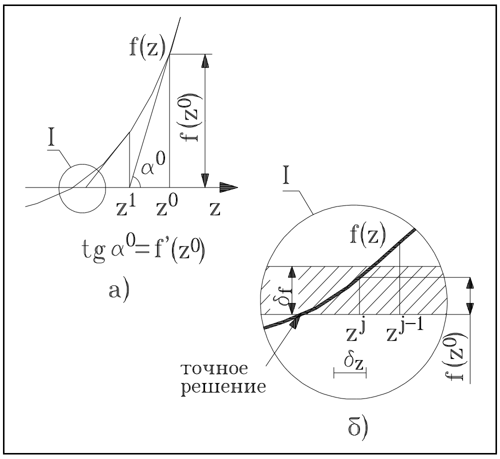


Fig. 1.5

where $f(z_{j-1})$ - the value of function $f(z)$ with $z=z_{j-1}$, f '(zj-1) - the value derived f (z)/dz with with z=zj-1;

e) checking on each iteration of the condition for the curtailment of iterations ([ris].1.5.[b]):

$$\left|z^j - z^{j-1}\right| = \left|\Delta z^j\right| \le \delta_z \qquad (1.15)$$

$$\left|f(z^j)\right| \le \delta_f$$

where δ_f - the permissible discrepancy (deviation from zero) of the right side of equation (1.13); δ_z - the permissible value of a difference in the solution on two adjacent iterations;

4) checking limitation to the maximum permissible quantity of the iterations:

$$j \le j_{max} \qquad (1.16)$$

Geometrically the solution of equation (1.13) is reduced to finding of the abscissa of point of intersection with the axis z by the curve $f(z)$. On each $j-1$ of the iteration of Newton's method the solution of this problem is substituted by finding point of intersection with tangent to the curve $f(z)$ with the z axis, in this case the tangent is built for $z=z_{j-1}$.

We return to the numerical solution of equation (1.12). After designating $z=v_1$, we have:

$$\alpha\, z|z| + \beta z + \gamma = 0 \qquad (1.17)$$

or

$$f(z) = 0,$$

where

$$f(z) = \alpha\, z|z| + \beta z + \gamma \qquad (1.18)$$

For solving equation (1.17) the expression for the derivative $f(z)$ will be required us by Newton's method:

$$f'(z) = 2\alpha\,|z| + \beta \qquad (1.19)$$

Let us assign initial data in order to calculate the values of coefficients $\alpha, \beta,$ τηε γ of equation (1.17):

k = of 20000 N/m,

$$\boxed{\mu = 1000\,\frac{Нс^2}{м^2}},$$

$\boldsymbol{m}$ **= of 0.1** ***kgf*****,**

$\boldsymbol{Q}$ **= 1000,** $\boldsymbol{T}$ **= 0.2p,** $\boldsymbol{F}$ **= of 1000sin 10t,**

Initial conditions and the step of the integration:

$$\boxed{x_0 = 0\,,\ \ v_0 = 0\,,\ \Delta t_1 = 0.001с}$$

Then, according to (1.12)

$$\boxed{\alpha = 100}$$

$$\boxed{\beta = \frac{200000*0.001}{2} + \frac{0.1}{0.001} = 11}$$

$$\boxed{\gamma\ = 200000 + 20000\frac{0*0.001}{2} - \frac{0.1*0}{0.001} - 1000\sin 0.01 = -1}$$

Thus,

$$\boxed{f(z) = 1000z|z| + 110z - 1} \tag{1.20}$$

$$\boxed{f'(z) = 2000|z| + 11} \tag{1.21}$$

Let us assign the values of the permissible errors for checking conditions (1.15):

$$\boxed{\delta_z = 0.001}$$

$$\boxed{\delta_f = 0.1} \tag{1.22}$$

The maximum permissible quantity of iterations let us take as equal **shch**.

Let us select initial approximation to the solution

z0 = **0**

<u>First iteration.</u>

$$\boxed{z^1 = z^0 + \Delta z^1}$$

$$\Delta z^1 = -\frac{f(z^0)}{f'(z^0)} = -\frac{1000\cdot 0\cdot|0| + 110\cdot 0 - 10}{2000|0| + 110} = 0.0909$$

$$z^1 = 0 + 0.0909 = 0.0909$$

Checking the completion of the iterations:

$$|\Delta z^1| > \delta_z$$

$$f(z^1) = 1000\cdot 0.0909\cdot|0.0909| + 110\cdot 0.0909 - 10 = 8.26$$

$$|f(z^1)| > \delta_f$$

Passage to the following iteration.

<u>Second iteration.</u>

$$z^2 = z^1 + \Delta z^2$$

$$\Delta z^2 = -\frac{f(z^1)}{f'(z^1)} = -\frac{8.26}{2000\cdot 0.0909 + 110} = -0.0283$$

$$z^2 = 0.0909 - 0.0283 = 0.0626$$

Checking the completion of the iterations:

$$|\Delta z^2| > \delta_z$$

$$f(z^2) = 1000\cdot 0.0626\cdot|0.0626| + 110\cdot 0.0626 - 10 = 0.80$$

$$|f(z^2)| > \delta_f$$

Passage to the following iteration.

<u>Third iteration.</u>

$$\Delta z^3 = -0.0034$$

$$z^3 = 0.0591$$

$$\left|\Delta z^3\right| > \delta_z$$

$$\left|f(z^3)\right| = 0.012 \delta_f$$

Fourth iteration.

$$\Delta z^4 = -0.0000$$

$$z^4 = 0.0591$$

$$\left|\Delta z^4\right| < \delta_z$$

$$\left|f(z^4)\right| = 0.0006 \delta_f$$

Both conditions (1.15) are satisfied, limitation (1.16) is not exceeded. The solution is achieved. We calculated the value of speed for moment of time ***t1***, after obtaining

$$v_1 = 0.05913 \ м/c$$

After using formulas (1.10), let us determine the values of acceleration and displacement for the same moment of time.

$$a_1 = \frac{0.05913 - 0}{0.001} = 59.13 \ м/c^2$$

$$x_1 = 0 + \frac{0 + 0.05913}{2} * 0.001 = 2.96\, e - 5 \ м$$

The solution for moment of time ***t1*** is obtained. Let us make one additional step on the time in order to illustrate now the selection of the value of step. Equations (1.9) - (1.12) are valid for any moment of time taking into account the corresponding replacement of subscripts. For is 2nd GO of step on the time we have:

$$kx_2 + \mu \ v_2\left|v_2\right| + ma_2 - Q \sin\frac{2\pi}{T} t_2 = 0$$

$$x_2 = x_1 + v_1 \Delta t_2 + a_2 \frac{\Delta t_2^2}{2} \qquad (1.23)$$

$$v_2 = v_1 + a_2 \Delta t_2$$

Just as at the first step, we reduce this system to one equation relative to the speed:

$$\boxed{\alpha\, v_2|v_2| + \beta\ v_2 + \gamma\ = 0},$$

where

$$\boxed{\alpha\ = \mu}$$

$$\boxed{\beta\ = \frac{k\Delta t_2}{2} + \frac{m}{\Delta t_2}}$$
$$\boxed{\gamma\ = kx_1 + k\frac{v_1\Delta t_2}{2} - \frac{mv_1}{\Delta t_2} - Q\sin\frac{2\pi}{T}t_2} \qquad (1.24)$$

The value of step **Δτ** $_2$ we preliminarily take as equal **Δτ** $_1$, i.e., **0.001** ***s.*** then, taking into account initial data and obtained at the first step of the solution, it is possible to calculate coefficients **α, β, τηε γ**:

$$\boxed{\alpha\ = 1000}$$

$$\boxed{\beta\ = \frac{20000*0.001}{2} + \frac{0.1}{0.001} = 110}$$

$$\boxed{\gamma\ = 20000*2.96e-5\ + 20000\frac{0.05913*0.001}{2} - \frac{0.1*0.05913}{0.001} -}$$
$$\boxed{-1000\sin\left(\frac{2\pi}{0.2\pi}*0.002\right) = -24.7}$$

We again have the nonlinear equation:

$$\boxed{1000v_2|v_2| + 110v_2 - 24.7 = 0}, \qquad (1.25)$$

which we solve by Newton's method.

In this place the smooth alliteration of our computations must be interrupted and focused special attention on the selection of initial approximation to the solution in the algorithm of Newton's method.

For the initial approximation to the solution let us accept such value of speed, which a body would have at the moment of time ***t2***, if the acceleration of body from moment of time ***t2*** did not change, i.e., we consider that

$$\boxed{v_2^0 = v_1 + a_1\Delta t_2} \qquad (1.26)$$

This the so-called explicit step (or forecast), when into formula for the speed enters already known acceleration. The velocity, obtained by explicit step, will use we not only as initial approximation in Newton's method, but also with the estimate of the magnitude of the selected step on the time.

Thus, initial approximation (forecast):

$$\boxed{v_2^0 = 0.05913 + 59.13 * 0.001 = 0.11826}$$

Omitting the detailed computations (they they are analogous to those given for the first step on the time), the iterations of Newton's method lead to the following sequence of values:

the initial approximation: $\boxed{v_2^0 = 0.11826}$

the first iteration: $\boxed{v_2^1 = 0.11172}$

the second iteration: $\boxed{v_2^2 = 0.11159}$, the solution is achieved.

We obtained that with the value of step τηε Δτ $_{\text{of } 2=0.001}$, speed for moment of time ***t2***

$$\boxed{v_2 = 0.11159 \ \text{м} / c}$$

Time is alien to estimate a error in the made step on the time.

A error in the method of integration on ***i- m*** step, called <u>local error</u>, we will evaluate according to the following formula:

$$\boxed{lp_i = \left| \frac{v_i^p - v_i^c}{2} \right|}, \quad (1.27)$$

where ***vip*** - explicit forecast of velocity on ***i- m*** step, determined by the formula

$$\boxed{v_i^p = v_{i-1} + a_{i-1} \Delta t_i}, \quad (1.28)$$

vic - value of speed, which we obtained the as a result iterative solution, using the implicit formula

$$\boxed{v_i^c = v_{i-1} + a_i \Delta t_i} \quad (1.29)$$

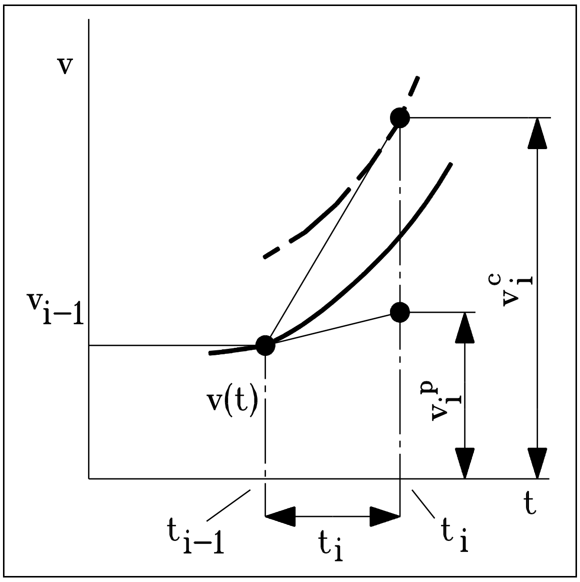


Fig. 1.6

Let us note that relationship (1.28) already adapted by us with the selection of initial approximation to the solution in the algorithm of Newton's method (look dependence (1.26)).

The calculation of speeds according to formulas (1.28) and (1.29) and the essence of the estimation of local error according to formula (1.27) explains Fig. 1.6.

At the moment of time ***ti -1*** we be situated at the point ***of vi-1***. If for enumerating the value ***vi*** we will use explicit formula (1.28), then the point ***of vi=vip*** will lie on the tangent, carried out to the curve ***v (t)*** at point ***ti -1***, since ***ai-1*** is a slope tangent of this tangent to the X-axis.

With the calculation ***vi*** with the use of formula (1.29) we need value ***ai***, i.e., the rate of change, carried out to the curve ***v (t)*** already at point ***ti***. Since at the moment of time ***ti -1*** we nothing know about the behavior of function ***v (t)*** with ***t=ti*** and tangent to the curve ***v (t)*** at point ***ti*** also conduct we cannot, then we calculate ***vi =vic*** not directly according to formula (1.29), but by the method of joint solution of system of equations (1.7), where it enters and relationship (1.29). In this case for us it is necessary to consecutively approach the solution (i.e., to ***vic***) for several Newtonian iterations.

Figure 1.6. shows that the explicit forecast ***of vip*** and the corrected solution ***of vic*** lie on the different sides from the curve ***v (t)***, passing through the point ***ti -1***. The greater the difference between ***vic*** and ***vip***, the stronger at the current step differs the graph of speed from the straight line and the higher the error in the integration at the step. Figure also makes it possible to understand that the decrease of the value of step ***ti*** leads to the decrease of the local error, evaluated according to formula (1.27), since decreases the divergence of the values ***of vip*** and ***vic***.

The calculation of local error is important to us not so much by itself, as as the means, which makes it possible to estimate the acceptability of the made step on the time and to recommend the value of the following step.

The mechanism of the determination of the value of step, on the basis of the criterion of local error, is sufficiently simple. The value of the maximum permissible local error at the step of integration is given $\boxed{\delta_l}$. According to the results of sequential i- ***GO*** of step the values of the permissible () $\boxed{\delta_l}$ and actually obtained local error are compared ($\boxed{lp_i}$). If $\boxed{lp_i \le \delta_l}$, then the made step is recognized as successful. Passage to the following step on the time is accomplished; its value for the one-step methods of integrating the first order of accuracy, to which correspond formulas (1.7) utilized by us, is selected on the dependence:

$$\boxed{\Delta t_{i+1} = c\Delta t_i \sqrt{\frac{\delta_l}{lp_i}}}\ , \quad (1.30)$$

where **Δτι** - value of the perfect step on the time,
Δτι+1– the recommended value of the following step,
c - correction factor, ***c < 1***.

But if $\boxed{lp_i > \delta_l}$, then the value of the made step **Δτι** is too great and does not ensure the required accuracy. Therefore it is necessary to conduct calculation on ***i- m*** step again, using with the reduced value **Δτι**. In this case for the selection of the value **Οφ δτι** also is used formula (1.30), only obtained on it value of step is used not for the following ***(i+1)*** - GO of step, but for the repeated calculation on current i- ***m*** step.

Returning for the investigated example of the numerical solution of equation (1.6), let us conduct for it is 2nd GO of the step of integration the estimation of local error and value of step.

In the course of computation we obtained the values:

$$\boxed{v_2^p = 0.11826}$$

$$\boxed{v_2^c = 0.11159}$$

Local error at the step:

$$\boxed{lp_2 = \left|\frac{0.11826 - 0.11159}{2}\right| = 0.00333}$$

After accepting the permissible error at the step:

$$\boxed{\delta_l = 0.001},$$

we are forced to establish that $\boxed{lp_2 > \delta_l}$ the, i.e., executed step with the value **τηε Δτ** ***of 2=0.001*** does not ensure the required accuracy of results and necessary to repeat calculation on is 2nd m step with the reduced value **Δτ** $_2$. The recommended value **Δτ** $_2$ for the repeated calculation let us determine with the aid of formula (1.30), using a coefficient ***of c=0.8***:

$$\boxed{\Delta t_2 = 0.8*0.001*\sqrt{\frac{0.001}{0.00333}} = 0.438e-3}$$

The results of repeated calculation with the step **Δτ** $_2$ = ***0.438e- e*** give the following values of the forecast of speed, corrected solution and local error:

$$\boxed{v_2^p = 0.08505}$$

$$\boxed{v_2^c = 0.08509}$$

$$\boxed{lp_2 = \left|\frac{0.08505 - 0.08509}{2}\right| = 0.00002}$$

Since the obtained now value $\boxed{lp_2 < \delta_l}$, the second step can be considered successful from the point of view of the assigned accuracy of the solution. Now let us supplement the calculated value of the speed ***of v2=0.08509 m/s*** with the values of acceleration ***a2*** and of displacement ***x2***, after using the formulas of connection (1.7):

$$a_2 = \frac{v_2 - v_1}{\Delta t_2} = \frac{0.08509 - 0.05913}{0.438e-3} = 59.21 \ м/c^2$$

$$x_2 = x_1 + v_1 \Delta t_2 + a_2 \frac{\Delta t_2^2}{2} = 2.96e-5 \ + \ 0.05913*0.438e-3 \ +$$

$$+59.21 \frac{(0.438e-3)^2}{2} = 6.12e-5 \ м$$

Up to the present moment we obtained numerical solution for two points of the temporary axis:

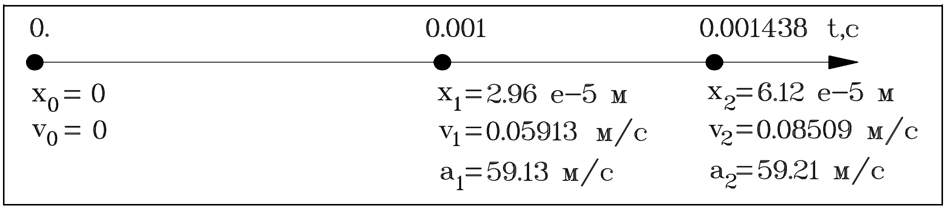


Fig. 1.7

Following the given algorithm, it is possible to continue calculation and to obtain the solution for entire time interval, which interests researcher.

Before summing up the first sums, would be desirable to return to Fig. 1.6. for some explanations. At the moment of time ***ti -1*** we be situated at the point ***of vi-1***. Through it the curve ***v*** passes ***(t)***. It is the so-called integral curve for moment of time ***ti -1***, i.e., the graph of speed, which corresponds to the exact solution of equation (1.6) with the initial condition

$$v\big|_{t=t_{i-1}} = v_{i-1} \tag{1.31}$$

Since we solve equation (1.6) approximately, actually on each i- ***m*** the step of numerical integration for the time we pass with one integral curve, which satisfies initial condition (1.31), to another integral curve, which is already the exact solution of equation (1.6) with the initial condition

$$v\big|_{t=t_i} = v_i \tag{1.32}$$

(in Fig. 1.6. integral curve for ***t=ti*** it is designated by dotted line).

Therefore as the result of numerical solution serve that broken, passing through the totality of the integral curves, each of which is the exact solution of equation (1.6) with the initial conditions, determined by numerical solution at the current step (Fig. 1.8).

We sum up the basic moments, essential from the point of view of the numerical analysis of the example examined.

The sequence of our actions was reduced to the following:

1. They formed the differential equation, which describes the behavior of the system:

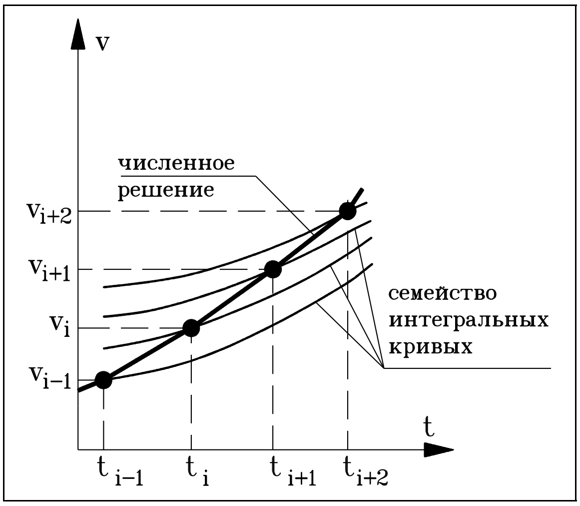


Fig. 1.8

$$kx + \mu \frac{dx}{dt}\left|\frac{dx}{dt}\right| + m\frac{d^2x}{dt^2} - Q\sin\frac{2\pi}{T}t = 0$$

With the formation of equation 2-1 Newton's law, which is been one of the methods of the recording of the condition of dynamic equilibrium, were used.

2. They represented the obtained equation in the form, that not containing clearly differential relationships, after writing down the latter separately:

$$kx + \mu\, v|v| + ma - Q\sin\frac{2\pi}{T}t = 0$$

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

e. Were replaced the differential linkage between ***x, v*** and ***a*** with the algebraic equations of relation, valid for the selected method of integration, after reducing thus the task of obtaining solving the in the form continuous functions to the task of finding the set of the values of unknown function at the isolated points of the temporary axis:

$$kx_i + \mu\, v_i|v_i| + ma_i - Q\sin\frac{2\pi}{T}t_i = 0$$

$$x_i = x_{i-1} + v_{i-1}\Delta t_i + a_i\frac{\Delta t_i^2}{2}$$

$$v_i = v_{i-1} + a_i\Delta t_i$$,

where **Δτι** - value i- GO of step on the time (**Δτι** = ***ti - ti -1***);

xi, vi, ai - value ***x, v*** and ***a*** with ***t=ti***.

4. The obtained system they reduced to one equation, after expressing ***xi, ai*** through ***vi***:

$$\boxed{\alpha \ v_i |v_i| + \beta \ \ v_i + \gamma \ = 0}$$

Thus, at each step on the time calculation was reduced to the solution of the nonlinear algebraic equation of form ***f (z) =0***, where ***z=vi***.

shch. The solution of nonlinear equation was carried out by Newton's method. This is the iterative numerical method (solution it is obtained approximated, with the predetermined accuracy, for several passages). For obtaining the solution on each passage it is necessary to calculate the values of function ***f (z)*** and by its derivative ***df (z)/dz***. We determine initial approximation to the solution, using a formula of explicit forecast.

'. The accuracy of numerical integration for the time was evaluated via the control of the local error at the step of integration, which depends on a difference in the explicit and implicit solution. With the unsatisfactory value of local error was repeated the calculation at the current step with the reduced value of step **Δτι**.

". If local error at the step is stale in the limits of that permitted, then considered step successful and, using the calculated value of speed ***vi***, were calculated acceleration ***ai*** and displacement ***xi*** over the equations of relation, valid for the selected method of integration.

8. The value of sequential step on the time was selected, on the basis of the relationship of the permissible and actually obtained local error at the current step of integration.

Based on this simple example we wanted to sufficiently designate the canvas of numerical solution, by which adheres to the algorithm of computational nucleus ***PRADIS*** by large smears. It is natural that the mass of most important questions remained out of the region of examination. To many of them we will return later, explanations on another better to obtain in the specialized literature, references on which with each opportunity we will give.

We hope that the given example makes it possible to understand the essence of the numerical solution of the differential equation of motion of body, formed in accordance with the design diagram accepted. However, it must be noted, that very formation of differential equation was conducted "by hand" and some questions in the course of computation were also solved nonformally (for example, the analytical determination of the form of the function, which is been derivative ***df (z)/dz*** in the algorithm of Newton's method). Therefore we continue the examination of methods and algorithms ***PRADIS*** from the explanation of the principles <u>of the automatic formation</u> of the system of the differential equations (for the considered example - one equation), which describe the behavior of the object being investigated.

2. Mechanism of the formation of the mathematical model

Let us return to the examination of system to [ris].1.1.

Let us rewrite again the equation of the equilibrium:

$$\boxed{F^c + F^y + F^в + F^u = 0} \qquad (2.1)$$

Since with the numerical integration we obtain the solution at the isolated points of temporary axis, for each i- ***GO*** of moment of time equation (2.1) can be recorded in the form:

$$\boxed{F_i^c + F_i^y + F_i^в + F_i^u = 0}, \quad (2.2)$$

where

$$\boxed{F_i^c = -Q\,sin\frac{2\pi}{T}t},$$

$$\boxed{F_i^y = kx_i}, \quad (2.3)$$

$$\boxed{F_i^в = \mu\ v_i|v_i|},$$

$$\boxed{F_i^u = ma_i}$$

We consider also that for ***i- GO*** of moment of time the values ***xi, vi, ai*** are connected with equations (1.7), which in view of their use in our further computations let us reproduce again:

$$\boxed{x_i = x_{i-1} + v_{i-1}\Delta t_i + a_i\frac{\Delta t_i^2}{2}} \qquad (2.4)$$

$$\boxed{v_i = v_{i-1} + a_i\Delta t_i}$$

You will focus attention, that equations (2.3) are differed from analogous expressions (1.3) in terms of sign. This connected with the fact that in ***PRADIS*** with the examination of the conditions of equilibrium are summarized the efforts, which act from the side of system to the elements, but not effort from the side of elements as this have accepted we with the selection of positive direction for the forces in accordance with Fig. 1.3.

It would be possible to obtain the equation of form (1.8) by substitution (2.3) in (2.2), but we this make will not be, since we should form and analyze mathematical model on the universal algorithm. We thus, have sufficiently universal equation of the equilibrium of form (2.2), valid for each i- ***GO*** of moment of time. Let us note that the passage from writing of the equation of equilibrium in the form (2.1) to the record in the form (2.2) marked with itself a qualitative change in the type of equation. If relationship (2.1) is differential equation (since the entering it dependences for the forces use derived displacements over the time), relationship (2.2) there is a already simply algebraic nonlinear equation (since the connection between ***xi, vi, ai*** is determined by the algebraic equations (2.4)). But in the form (2.2) it is possible to enter also with the nonlinear equation, as we enter with equation (1.12), namely: to decide by his method of Newton.

We have

$$\boxed{f(z)=0}, \quad (2.5)$$

where

$$\boxed{f(z)=F_i^c+F_i^y+F_i^{в}+F_i^{u}}$$

Variable ***z*** we can designate any of the components ***xi, vi*** or ***ai***, since they are interconnected by relationships (2.4). Let us accept, as before ***z=vi***.

On each iteration, in accordance with formulas (1.14), we should calculate value ***f (z)*** and by its derivative ***df (z)/dz***.

Calculation ***f (z)*** is reduced to the summing up of the instantaneous values of forces with the instantaneous values ***xi, vi, ai*** (***i*** - number of step on time, ***j*** - number of iteration according to Newton). Actually, the computable value ***f (z)*** is the error in the fulfillment of conditions of equilibrium, which by Newtonian iterations must be “driven in” within the permissible limits.

Now let us paint derivative ***df (z)/dz***.

$$\boxed{\frac{df(z)}{dz}=\frac{d\left(F_i^c+F_i^y+F_i^{в}+F_i^{u}\right)}{dz}=\frac{dF_i^c}{dz}+\frac{dF_i^y}{dz}+\frac{dF_i^{в}}{dz}+\frac{dF_i^{u}}{dz}} \quad (2.6)$$

Acting strictly in the science, each of the derivatives in expression (2.6) we must represent as the derivative of complex function.

$$\boxed{\frac{dF_i^c}{dz}=\frac{\frac{\partial F_i^c}{\partial x_i}dx_i+\frac{\partial F_i^c}{\partial v_i}dv_i+\frac{\partial F_i^c}{\partial a_i}da_i}{dz}=\frac{\partial F_i^c}{\partial x_i}\frac{dx_i}{dz}+\frac{\partial F_i^c}{\partial v_i}\frac{dv_i}{dz}+\frac{\partial F_i^c}{\partial a_i}\frac{da_i}{dz}}$$

Since ***z=vi***, then

$$\boxed{\frac{dF_i^c}{dz}=\frac{dF_i^c}{dv_i}=\frac{\partial F_i^c}{\partial x_i}\frac{dx_i}{dv_i}+\frac{\partial F_i^c}{\partial v_i}\frac{dv_i}{dv_i}+\frac{\partial F_i^c}{\partial a_i}\frac{da_i}{dv_i}} \quad (2.7)$$

It is analogous:

$$\boxed{\frac{dF_i^y}{dz}=\frac{dF_i^y}{dv_i}=\frac{\partial F_i^y}{\partial x_i}\frac{dx_i}{dv_i}+\frac{\partial F_i^y}{\partial v_i}\frac{dv_i}{dv_i}+\frac{\partial F_i^y}{\partial a_i}\frac{da_i}{dv_i}}$$

$$\boxed{\frac{dF_i^{в}}{dz}=\frac{dF_i^{в}}{dv_i}=\frac{\partial F_i^{в}}{\partial x_i}\frac{dx_i}{dv_i}+\frac{\partial F_i^{в}}{\partial v_i}\frac{dv_i}{dv_i}+\frac{\partial F_i^{в}}{\partial a_i}\frac{da_i}{dv_i}} \quad (2.7a)$$

$$\frac{dF_i^u}{dz}=\frac{dF_i^u}{dv_i}=\frac{\partial\ F_i^u}{\partial\ x_i}\frac{dx_i}{dv_i}+\frac{\partial\ F_i^u}{\partial\ v_i}\frac{dv_i}{dv_i}+\frac{\partial\ F_i^u}{\partial\ a_i}\frac{da_i}{dv_i}$$

It is utilized the equation of relation (2.4) for obtaining the dependences ***ai*** and ***xi*** on ***vi***:

$$a_i=\frac{v_i-v_{i-1}}{\Delta t_i}$$, (2.8)

$$x_i=x_{i-1}+\frac{v_i+v_{i-1}}{2}\Delta t$$

We differentiate expressions (2.8) on ***vi***

$$\frac{dx_i}{dv_i}=\frac{\Delta t_i}{2}$$

$$\frac{dv_i}{dv_i}=1$$ (2.9)

$$\frac{da_i}{dv_i}=\frac{1}{\Delta t_i}$$

Now let us calculate partial derivatives in expressions (2.7), (2.7[a]), using dependences (2.3):

<u>For the force ***Of fi[s]***:</u>

$$\frac{\partial\ F_i^c}{\partial\ x_i}=0$$ (***Fi[s]*** it does not depend on the displacement of body)

$$\frac{\partial\ F_i^c}{\partial\ v_i}=0$$ (***Fi[s]*** it does not depend on the speed of body) (2.10)

$$\frac{\partial\ F_i^c}{\partial\ a_i}=0$$ (***Fi[s]*** it does not depend on the acceleration of body)

<u>For the force ***To fi[u]***:</u>

$$\frac{\partial\ F_i^y}{\partial\ x_i}=k$$

$$\frac{\partial\ F_i^y}{\partial\ v_i}=0$$ (***To fi[u]*** it does not depend on the speed of body) (2.11)

$$\boxed{\frac{\partial F_i^y}{\partial a_i} = 0}$$

(***To fi[u]*** it does not depend on the acceleration of body)

For the force ***Of fi[v]***:

$$\boxed{\frac{\partial F_i^e}{\partial x_i} = 0}$$

(***Fi[v]*** it does not depend on the displacement of body)

$$\boxed{\frac{\partial F_i^e}{\partial v_i} = 2\mu\ |v_i|} \qquad (2.12)$$

$$\boxed{\frac{\partial F_i^d}{\partial a_i} = 0}$$

(***Fi[v]*** it does not depend on the acceleration of body)

For the force ***Of fi[i]***:

$$\boxed{\frac{\partial F_i^u}{\partial x_i} = 0}$$

(***Fi[i]*** it does not depend on the displacement of body)

$$\boxed{\frac{\partial F_i^u}{\partial v_i} = 0}$$

(***Fi[i]*** it does not depend on the speed of body) (2.13)

$$\boxed{\frac{\partial F_i^u}{\partial a_i} = m}$$

We substitute the obtained values of particular derivatives of efforts and the values of coefficients (2.9) in formulas (2.7), (2.7[a]):

$$\boxed{\frac{\partial F_i^c}{\partial z} = 0},$$

$$\boxed{\frac{\partial F_i^y}{\partial z} = \frac{k\Delta t}{2}}, \quad (2.14)$$

$$\boxed{\frac{\partial F_i^e}{\partial z} = 2\mu\ |v_i|},$$

$$\boxed{\frac{\partial F_i^u}{\partial z} = \frac{m}{\Delta t_i}}$$

We summarize the terms of formula (2.6):

$$\boxed{\frac{\partial\, f(z)}{\partial\, z} = \frac{k\Delta t_i}{2} + 2\mu\,|v_i| + \frac{m}{\Delta t_i}} \qquad (2.15)$$

Comparing result with previously obtained formula (1.19), for which the coefficients are taken from relationships (1.12), it is possible to establish that they are similar.

Now, having the capability to calculate ***f (z)*** and ***df (z)/dz,*** to continue calculation according to algorithm described earlier does not present labor. However, as spoke one their heroes Of [tolkiena], "situation at the present moment can be by that requiring some explanations". After being torn through the paling of total and particular derivatives, we obtained the same result as earlier, but patience in the reader these computations for sure fairly of [poubavili]. By what acquisitions do redeem these labor expenses?

1. Not at all it was necessary to extract differential equation of motion.

2. The expanded form of the nonlinear equation of form (1.12), to solution of which is reduced the calculation at each step on the time, also proved to be uncalled-for.

e. Functional dependence for the derivative ***df (z)/dz*** was not required.

If we attentively examine our reasonings, then we used the following information (list of th mathematical model of information, e necessary for the formation given below we will further call "***enumeration***"):

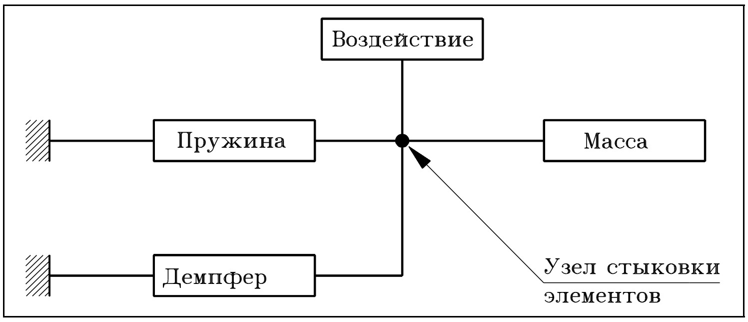


Fig. 2.1

1. Information about jointing of the elements of network (Fig. 2.1).

2. Condition of equilibrium of forces, recorded for i- GO of moment of time, and the being nonlinear algebraic equation relative to ***xi, vi, ai*** the form:

$$\boxed{\sum_{k=1}^{4} F_i^{(k)} = 0}\ , \quad (2.16)$$

where **4-** number of forces, which are converged in the unit of jointing (quantity of [stykujushchikhsja] branches of elements).

Equation (2.16) still is called ***topological***, since it is determined by topology, i.e., structure of connections in the diagram.

e. Expressions, which make it possible to define efforts in each element as the function of displacement, speed, acceleration and time - (2.3). This the so-called ***component*** equations, which describe the behavior of the separate component (element) of diagram.

4. Expressions for the particular derivatives of the efforts, which act in the element, on the displacement, the speed, to acceleration - see dependences (2.10) - (2.14).

shch. Algebraic equations of relation ***x, v, a*** for current time - the formula of the method of integration (2.4).

'. Indication, relative to which of the variables - ***x, v*** or ***a*** - to conduct the solution of nonlinear equation, i.e., which is selected as variable ***z*** of function ***f (z)*** in formula (2.5).

Of the enumerated information it is sufficient for the realization of the machine algorithms of the formation of the mathematical model of object.

So that in the reader it would not remain dark places and white spots, we again in more detail will pass on example already repeatedly dismantled at this document.

The thus, let there be technical system, processes in which require analysis. Design diagram corresponds [ris].1.1.

System it is necessary to present in the form the totality of the elements, [stykujushchikhsja] according to the general degrees of freedom (units). A quantity of degrees of freedom (units) in each element is determined by the variety of element (Fig. 2.2).

There is a concept ***of the model of element***. User gathers ***the model of system*** from the models of separate elements as toy in the children's designer. It (user) it must worry only the correctness of assembling, remaining questions of the formation of mathematical model - headache of the developers of software. Having before itself the design diagram (Fig. 1.1) and exarticulating from it elements (Fig. 2.2), user finds the models of the equivalent components in the library of the models of the elements of program set and describes the structure of the diagram being investigated.

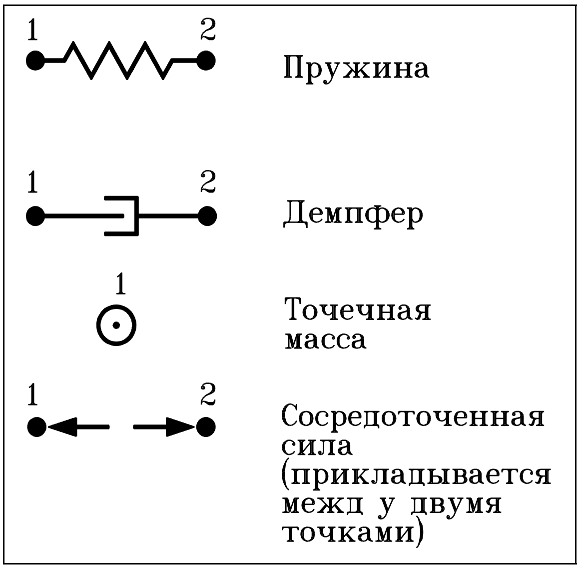


Fig. 2.2

The description of structure consists in the connection of elements according to the general degrees of freedom and the indication of the fixed units. For the completeness of picture let us give the piece of the text of the description of structure in the input language ***PRADIS***:

```
I FRAGMENT: Example

# BASE: 1

# STRUCTURE:
[Pruzhina]' K (1 2; Stiffness coefficient)
Nonlinear damper 'MUNL (1 2; Coefficient of viscosity)
Mass 'M (2; Mass of body)
Action 'FSIN (2 1; Q, T, initial phase)
```

By the preparation for the given description user reported to program set ***PRADIS*** entire necessary information on jointing of the elements of the network (see the point of 1 enumerations of the necessary for information for the formation mathematical model of object). User, in the first place, selected the models of the elements from the library of models - this of model **K, MUNL, M, FSIN**. In the second place, connected they properly, after applying action to the mass in unit **2**, to which it also joined the ends of spring and damper. Finally, described unit **1** as fixed, after fastening the thus free ends of spring and damper.

Let us examine now those actions of the programs, which make it possible as a whole to present the mechanism of the work of computational algorithm.

In the process of working the description of the structure of model is determined the dimensionality of system of equations, i.e., a quantity of units, in which must be satisfied the conditions of equilibrium. In the example in question two units, one of which is fixed. At the stage of the formation of mathematical model the structure of data will be prepared on both units; however, in the stage of calculation the equation, which corresponds to the fixed unit, is excluded from the examination, and all kinematic characteristics of the fixed unit (displacement, speed, acceleration) it is established in zero.

The stage of numerical integration is the sequence of the steps on the time, each of which is reduced to the solution of the nonlinear equation of the equilibrium of form (2.16). The information, given above in the points of 3-6 ***enumerations,*** is necessary for solving this equation.

Now time itself to focus attention on the model of elements and to explain, which their role in the computational algorithm. Input information for any model of element are:

- the constant list of the parameters of the model of element;

- the instantaneous values of displacements, speeds and accelerations of those units, with which this element is connected.

The model of element is obligated for current time to calculate according to these data:

1) the efforts, which act from the side of system to the elements, i.e., the vector of the efforts of the element (see the point e ***of enumeration***);

2) the partial derivatives of the computable efforts for displacements, speeds and accelerations of the units of element, i.e., the so-called Jacobi matrix (jacobian) the element (see the point of 4 ***enumerations***).

If element has ***N*** of degrees of freedom, then the length of the vector of the efforts of element also ***N***, and the jacobian of element has a length ***N* * *N* * e**.

How this does appear? For example, the developer of the binodal model of one-dimensional dimensionless inertia-free ideally elastic spring, which we is utilized in our example,
it realized the following dependences for the efforts and the jacobian of element (Fig. 2.3):

$$\boxed{F_1 = k(x_1 - x_2)},$$

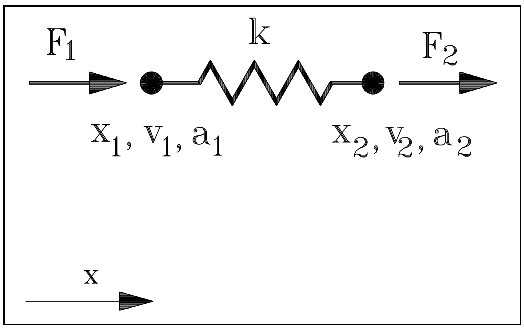


Fig. 2.3

$$\boxed{F_2 = k(x_2 - x_1)} \qquad (2.17)$$

$$\boxed{\frac{\partial F_1}{\partial x_1} = k}, \quad \boxed{\frac{\partial F_1}{\partial x_2} = -k} \qquad (2.18)$$
$$\boxed{\frac{\partial F_2}{\partial x_1} = -k}, \quad \boxed{\frac{\partial F_2}{\partial x_2} = k}$$

$$\boxed{\frac{\partial F_1}{\partial v_1} = \frac{\partial F_1}{\partial v_2} = \frac{\partial F_2}{\partial v_1} = \frac{\partial F_2}{\partial v_2} = 0} \qquad (2.19)$$

$$\boxed{\frac{\partial F_1}{\partial a_1} = \frac{\partial F_1}{\partial a_2} = \frac{\partial F_2}{\partial a_1} = \frac{\partial F_2}{\partial a_2} = 0} \qquad (2.20)$$

In accordance with the given dependences, for any moment of time the model of element in terms of tN instantaneous values of displacements, speeds and accelerations (although for the element in question are important only the displacements) e transmitted into it calculates the values of the efforts, which act on the ends of spring, and the value of particular derivatives of efforts for displacements, speeds and accelerations of both units. The vector of efforts consists their **2** elements, jacobian - of **12**.

Since in the design diagram of object in question unit 1 of spring is fixed, in this specific case from the entire information, computed by model and transferred “upward”, will be claimed only that part, which is connected with the loose second unit:

$$\boxed{F_2 = k(x_2 - x_1)} \qquad (2.21)$$

$$\boxed{\frac{\partial F_2}{\partial x_2} = k} \qquad (2.22)$$
$$\boxed{\frac{\partial F_2}{\partial v_2} = \frac{\partial F_2}{\partial a_2} = 0}$$

This information makes it possible to consider the contribution of spring during the solution of the nonlinear equation of form (2.5) on the algorithm, presented with the conclusion of relationships (2.6) - (2.15). The contribution of remaining elements (damper, mass, external action) is considered analogously.

In order to define concretely the aforesaid, let us continue the previously integration of a example initiated, after making sequential 3- 1 step on the time. In this case we will use the formal algorithm, which is been based on the sequence of calculations according to formulas (2.5) - (2.15).

Let us recall that according to the results is 2nd GO of step on the time (with ***t2=1.438e-3***) we obtained the following values of unknowns:

$$\boxed{x_2 = 6.12e-5\ м}$$

$$\boxed{v_2 = 0.08509\ м/с}$$

$$\boxed{a_2 = 59.21\ \ м/с^2}$$

The value of step was equal **Δt2=0.438** ***e-3***, th value of local error e obtained at the step composed ***lp2=0.000018***.

The recommended value **of Δt3** for the following step we determine from formula (1.30) taking into account ***of c=0.8*** and **δl = 0.001**:

Further we act according to the diagram, represented in the figures of 2.4[a]-2.4[v]. from Fig. of 2.4.[a] it follows that before ***i- m*** with step on the time must be known the values ***ti -1, xi-1, vi-1, ai-1,*** **Δti**. It is possible to ascertain that before beginning 3- GO of step we actually have available information about the values ***t2, x2, v2, a2,*** **Δt3**.

The details of the algorithm of the fulfillment of separate step let us get from Fig. of 2.4.[b].

1. We determine the values of the reduction coefficients of jacobian - $\boxed{\partial x_i/\partial z}$ $\boxed{\partial v_i/\partial z}$, $\boxed{\partial a_i/\partial z}$(see formulas (2.7), (2.7[a])), that depend on the value of step. Since during calculations at the first two steps for the basic variable we already accepted speed (i.e., ***z=vi***), summing up of jacobian is conducted through formulas (2.7), (2.7[a]), for which the reduction coefficients are calculated from dependences (2.9):

$$\boxed{\frac{dx_3}{dv_3} = \frac{\Delta t_3}{2} = \frac{2.63e-3}{2} = 1.31e-3}$$

$$\boxed{\frac{dv_3}{dv_3} = 1}$$

$$\boxed{\frac{da_3}{dv_3} = \frac{1}{\Delta t_3} = \frac{1}{2.63e-3} = 380.2}$$

Let us recall that relationships (2.9) are determined by the formulas of the method of integration (2.4).

2.Calculate the initial approximation to the solution by the formula of explicit forecast (1.28):

$$\boxed{v_3^0 = v_2 + a_2\Delta t_3 = 0.08509 + 59.21 * 2.63e-3 = 0.24081}$$

You will focus attention, that as the initial approximation must be calculated not only the value ***of vi0***, but also value ***of xi0, ai0***, necessary for the calculation in the models of elements.

Therefore:

$$\boxed{a_3^0 = a_2 = 59.21}$$

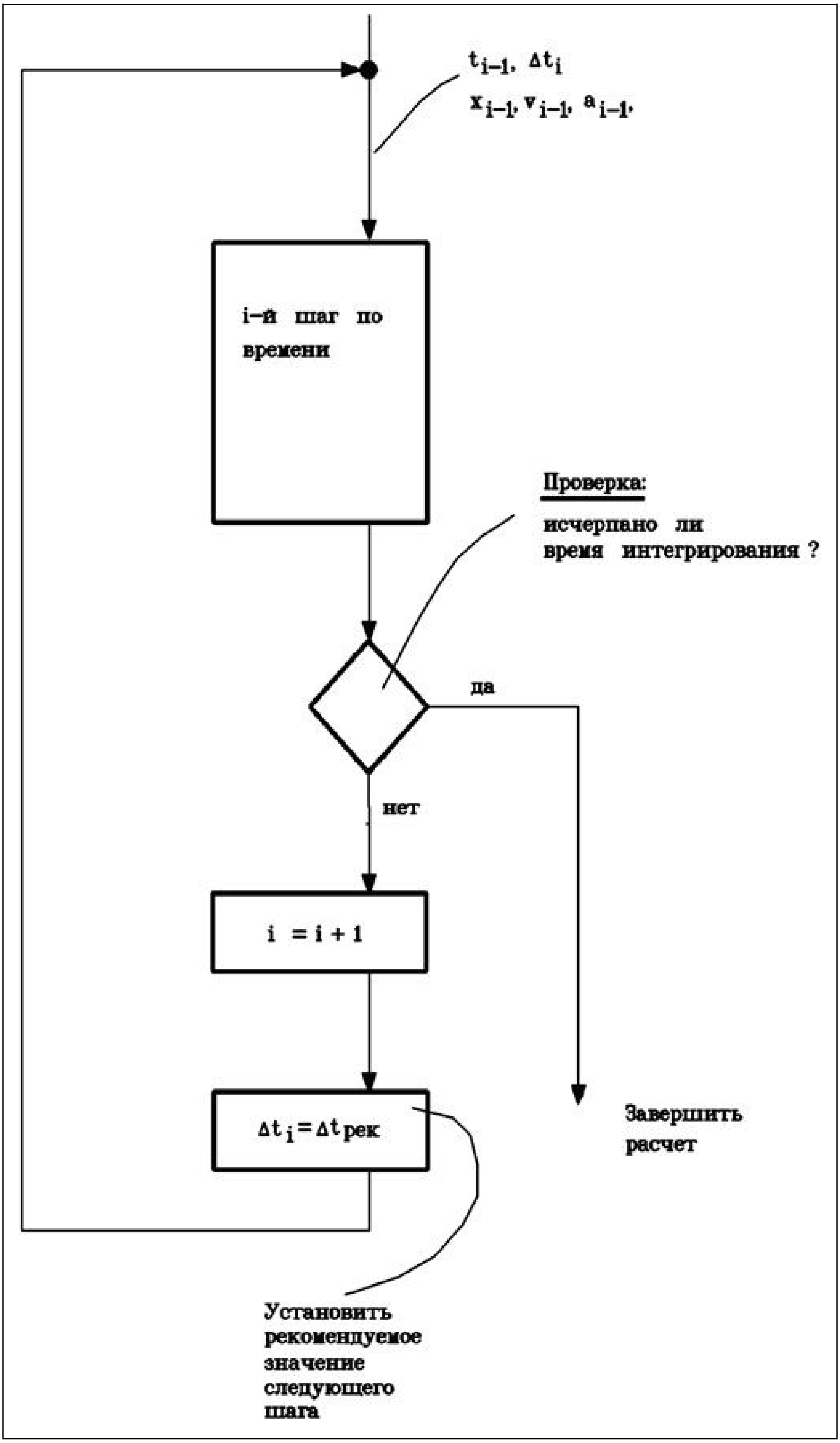
$$\boxed{x_3^0 = x_2 + v_2\Delta t_3 + a_3\frac{\Delta t_3^2}{2} = 6.12e-5\ + 0.08509 * 2.63e-3\ +}$$

$$\boxed{+59.21\frac{(2.63e-3)^2}{2}=46.5e-5}$$

e. After establishing the counter of iterations by equal to **1**, is realized the sequence of actions on **1-** 1 iteration of Newton (see Fig 2.4[v]).

4. Turning to the models of elements. Calculation of the vector of forces and jacobian of each element in terms of the instantaneous values ***of x30, v30, a30***.

At the present moment let us limit to data analysis only on the loose unit:



t_{i-1}, Δt_i
x_{i-1}, v_{i-1}, a_{i-1},
i-й шаг по времени
Проверка:
исчерпано ли время интегрирования ?
да
нет
i = i + 1
$\Delta t_i = \Delta t_{рек}$
Завершить расчет
Установить рекомендуемое значение следующего шага

Fig. of 2.4[a]

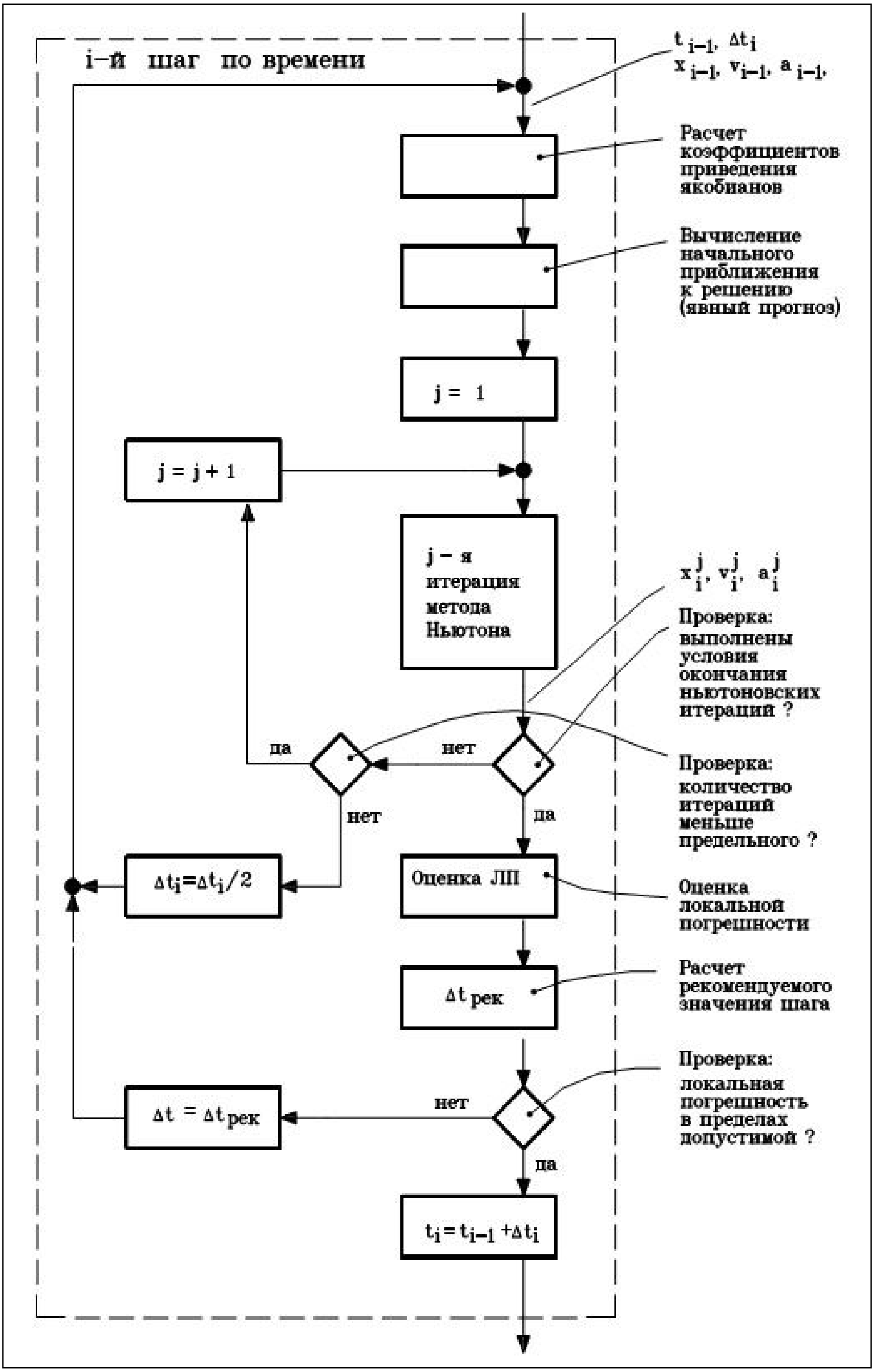

i–й шаг по времени
t_{i-1}, Δt_i
x_{i-1}, v_{i-1}, a_{i-1},
Расчет коэффициентов приведения якобианов
Вычисление начального приближения к решению (явный прогноз)
j = 1
j = j + 1
j – я итерация метода Ньютона
x_i^j, v_i^j, a_i^j
Проверка: выполнены условия окончания ньютоновских итераций ?
да
нет
нет
да
Проверка: количество итераций меньше предельного ?
$\Delta t_i = \Delta t_i / 2$
Оценка ЛП
Оценка локальной погрешности
$\Delta t_{рек}$
Расчет рекомендуемого значения шага
Проверка: локальная погрешность в пределах допустимой ?
$\Delta t = \Delta t_{рек}$
нет
да
$t_i = t_{i-1} + \Delta t_i$

Fig. of 2.4[b]

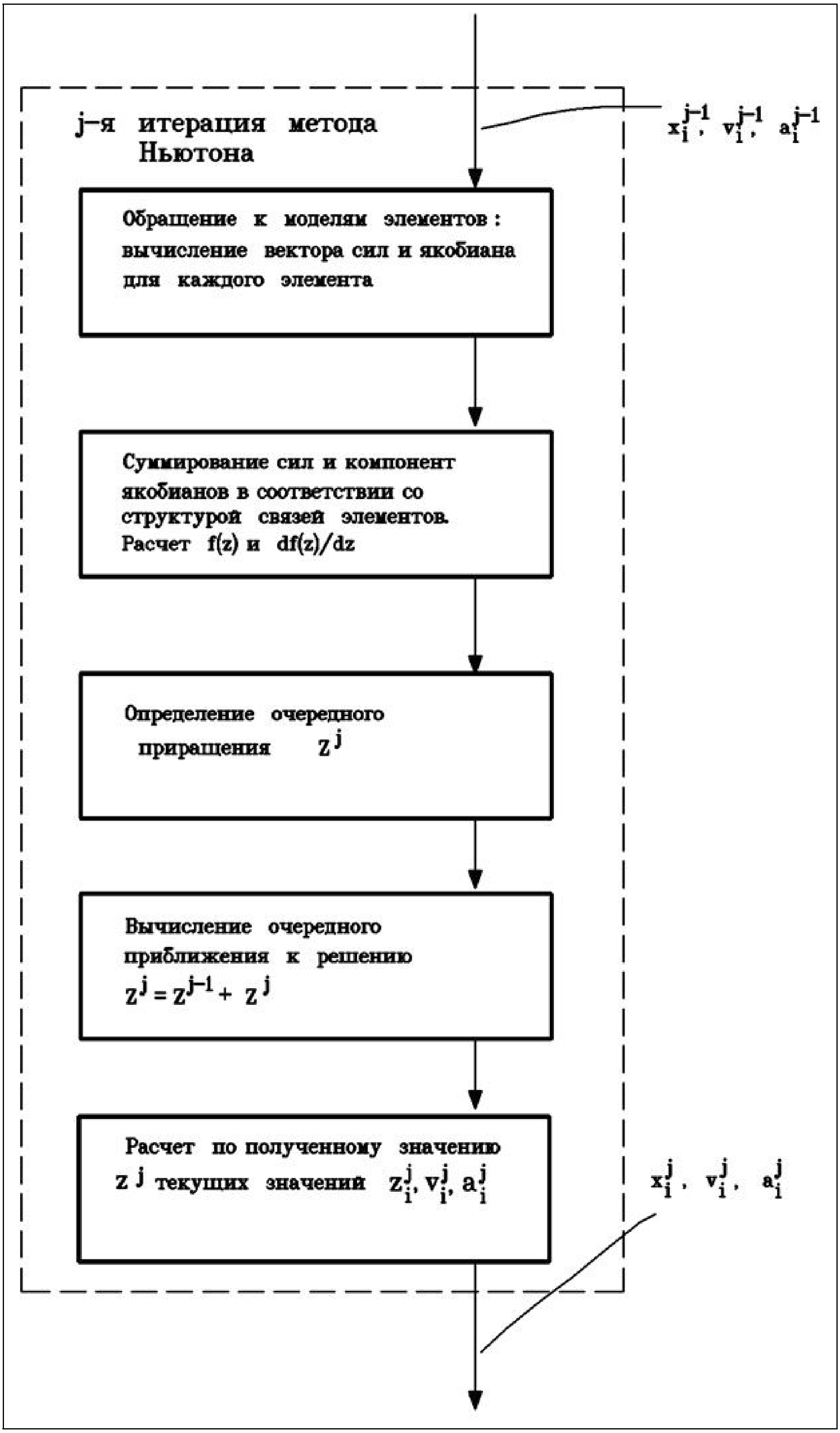

j–я итерация метода Ньютона
x_i^{j-1}, v_i^{j-1}, a_i^{j-1}
Обращение к моделям элементов : вычисление вектора сил и якобиана для каждого элемента
Суммирование сил и компонент якобианов в соответствии со структурой связей элементов. Расчет f(z) и df(z)/dz
Определение очередного приращения z^j
Вычисление очередного приближения к решению $z^j = z^{j-1} + z^j$
Расчет по полученному значению z^j текущих значений z_i^j, v_i^j, a_i^j
x_i^j, v_i^j, a_i^j

Fig. of 2.4[v]

Spring:

$$F^{y} = kx = 20000 * 46.5e - 5 = 9.3$$

$$\frac{\partial F^{y}}{\partial x} = k = 20000$$

$$\frac{\partial F^{y}}{\partial v} = \frac{\partial F^{y}}{\partial a} = 0$$

Damper:

$$F^{в} = \mu \ v|v| = 1000 * 0.24081 * |0.24081| = 58.0$$

$$\frac{\partial F^{в}}{\partial x} = 0$$

$$\frac{\partial F^{в}}{\partial v} = 2\mu \ |v| = 2 * 1000 * |0.24081| = 481.6$$

$$\frac{\partial F^{y}}{\partial a} = 0$$

Mass point:

$$F^{u} = ma = 0.1 * 59.21 = 5.9$$

$$\frac{\partial F^{u}}{\partial x} = \frac{\partial F^{u}}{\partial v} = 0$$

$$\frac{\partial F^{u}}{\partial a} = m = 0.1$$

Applicable force:

$$F^{c} = Q \sin \frac{2\pi}{T} t = -1000 \sin \frac{2\pi}{0.2\pi} (1.438e - 3 \ + 2.63e - 3) = -40.6$$

$$\frac{\partial F^{c}}{\partial x} = \frac{\partial F^{c}}{\partial v} = \frac{\partial F^{c}}{\partial a} = 0$$

shch. We summarize the forces, calculated in the models of elements and which are converged in the loose unit,

$$\boxed{\sum F = F^{y} + F^{\text{в}} + F^{u} + F^{c} = -40.6 + 9.3 + 58.0 + 5.9 = 32.6}$$

The obtained sum is the value of function ***f (z)*** on the current iteration (see expression (2.5)).

Let us calculate now ***df (z)/dz***, using relationships (2.6), (2.7), (2.7[a]):

$$\frac{df(z)}{dz} = \frac{dF_i^{c}}{dz} + \frac{dF_i^{y}}{dz} + \frac{dF_i^{\text{в}}}{dz} + \frac{dF_i^{u}}{dz}$$

$$\frac{dF^{c}}{dz} = 0*1.31e-3 + 0*1 + 0*380.2 = 0$$

$$\frac{dF^{y}}{dz} = 20000*1.31e-3 + 0*1 + 0*380.2 = 26.2$$

$$\frac{dF^{\text{в}}}{dz} = 0*1.31e-3 + 481.6*1 + 0*380.2 = 481.6$$

$$\frac{\partial F^{u}}{\partial z} = 0*1.31e-3 + 0*1 + 0.1*380.2 = 38.0$$

$$\frac{df(z)}{dz} = 0 + 26.2 + 481.6 + 38.0 = 545.8$$

6.[Opredeljaem] the increase :

$$\Delta z^{1} = \frac{f(z^{0})}{f'(z^{0})} = -\frac{32.6}{545.8} = -0.05973$$

". We calculate sequential approximation to the solution

$$z^{1} = z^{0} + \Delta z^{1} = 0.24081 - 0.05973 = 0.18108$$

8. In terms of the obtained value we refine the instantaneous values ***of x31, v31, a31***, using formulas (2.81):

$$v_3^{1} = z^{1} = 0.18108$$

$$a_3^{1} = \frac{v_3^{1} - v_2}{\Delta t_3} = \frac{0.18108 - 0.08509}{2.63e-3} = 36.5$$

$$x_3^{1} = x_2 + \frac{v_2 + v_3^{1}}{2}\Delta t_3 = 6.12e-5 + \frac{0.08509 + 0.18108}{2}*2.63e-3 = 41.1e-5$$

Calculations on the first iteration of Newton are finished.

9. We check the conditions of the completion of Newtonian iterations. Let us recall that earlier we accepted the following values of the permissible errors for checking conditions (1.15):

$$\boxed{\delta_z = 0.001}$$

$$\boxed{\delta_f = 0.1}$$

On the basis of these values, we conclude that

$$\boxed{\left|f(z^0)\right| > \delta_f}$$

$$\boxed{\left|\Delta z^1\right| > \delta_z}$$

Thus, Newtonian iterations at the current step on the time must be continued.

10. , before passing to the following iteration, we check, is not exhausted a maximum quantity of the iterations:

***j* = *1* < *of jmax* = shch**

11. We increase the counter of the number of the iterations:

***j* = *j* + 1 = 1 + 1 = 2**

12. We check the sequence of actions in Fig. of 2.4[v] for the second iteration of Newton's method. These actions will lead us to the following solution:

$$\boxed{f(z^1) = \sum F = -3.6}$$

$$\boxed{\Delta z^2 = -0.00858}$$

$$\boxed{x_3^2 = 39.8e-4}$$

$$\boxed{v_3^2 = 0.17250}$$

$$\boxed{a_3^2 = 33.2}$$

Checking the conditions of the end of iterations will show that the iterations are not yet finished:

$$\boxed{\left|f(z^1)\right| > \delta_f}$$

$$\boxed{\left|\Delta z^2\right| > \delta_z}$$

Checking:

***j* = 2 < *of jmax* = shch**

last obstacles from the way of fulfilling the sequential, third iteration are removed.

13. The third iteration will prove to be the latter. The following result will be obtained:

$$\left|f(z^2)\right| = \left|-0.07\right| < \delta_f$$

$$\left|\Delta z^3\right| = \left|-0.00018\right| < \delta_z$$

$$x_3^3 = 39.8e-4$$

$$v_3^3 = 0.17232$$

$$a_3^3 = 33.2$$

14. In accordance with the diagram of 2.4[b], after the successful completion of Newtonian iterations it is necessary to estimate the value of local error at the step of the integration

$$lp_3 = \left|\frac{v_3^p - v_3^c}{2}\right| = \left|\frac{0.24081 - 0.17232}{2}\right| = 0.034$$

15. We calculate th value of the following step e recommended on the criterion of local error.

Here should be put one stage direction. The practice of calculations showed that formula (1.30) was acceptable only in the specific range of the relationships **of δl/of *lpi***, namely - near one. With significant differences **in δl/of *lpi*** from one the recommended with formula (1.30) value of step is, as a rule, overstated leads to the unjustified loss of steps because of the noncompliance to the requirements of accuracy in the integration. Of this we will be convinced even now, since for the selection of the value of the current step used formula (1.30) with the relationship **of δl/*lpi*** = **0.001/0.000018 = 55.5**. As the consequence of this, the made step with the recommended with formula (1.30) value of the step **of Δt3=2.63e-3** led us to the result, when the comparison of that obtained (***lp3*** **= 0.034**) and maximum permissible (**δl=0.001**) local errors actually determines the need of repeating the calculations n 3. of m step with the reduced value of step.

We correct the rule of the selection of step on the criterion of local error. It appears as follows:

$$\Delta t_{рек} = \begin{cases} c * \Delta t_i \dfrac{\delta_l}{lp_i} & при \quad \dfrac{\delta_l}{lp_i} < 0.25 \\ c * \Delta t_i \sqrt[4]{\dfrac{\delta_l}{lp_i}} & при \quad \dfrac{\delta_l}{lp_i} > 0.25 \\ c * \Delta t_i \sqrt{\dfrac{\delta_l}{lp_i}} & при \quad 0.25 \le \dfrac{\delta_l}{lp_i} \le 7 \end{cases} \qquad (2.23)$$

Then, continuing the consideration of algorithm from the interrupted place, the recommended value of step for the repeated calculation n 3. of m step let us determine taking into account (2.23):

$$\frac{\delta_l}{lp_3} = \frac{0.001}{0.034} = 0.03$$

Since **δl/*of lp3* < 0.25**,

$$\Delta t_{рек} = c * \Delta t_3 * \frac{\delta_l}{lp_3} = 0.8 * 2.63e-3 * \frac{0.001}{0.034} = 0.061e-3\ c$$

16. We establish **Δt3= 0.061 ** e-3*** and we repeat the calculations n 3. of m step, beginning from point 1.

Repeated calculation with this value of step leads to the following results for moment of the time ***of t3=t2+ t3* = 1.499 ** e-3***:

$$x_3 = 6.64e-5\ м$$

$$v_3 = 0.08862\ м/c$$

$$a_3 = 58.1\ м/c^2$$

Local error in the limits of standard. Recommended value for the following step **of Δt[rek]=0.264 ** e-3[s].***

Calculation at the third step on the time is finished.

Basic, to what it would be desirable to focus attention on the completion of the selection of a example, this separation of the functions between strictly the program of integration and the programs of the realization of the models of elements. To program of integration, which works on the algorithm Fig. of 2.4[a]-2.4[v], generally speaking, nevertheless, what processes to integrate. Its dependence on the models of elements is reduced only to timely obtaining from them of the vectors of forces and matrices of jacobians. But this information reflects what properties of separate elements, the program of integration this concern must not. The models of elements, in turn, have their level of the independence of information with the completely outlined responsibilities before “the tops”. I.e., the physical properties of the separate element of object are reflected in the component equations in the level of the model of element, and the program of integration works at the level of the equations of the equilibrium of flows, without concerning, from what relationships the components of these flows are calculated. This differentiation of functions determines the universality of computational nucleus ***PRADIS*** the, i.e., possibility in principle of calculating any objects, processes in which are subordinated to equilibrium law of flow variables (equilibrium of forces, electrical and heat fluxes, fluid flow rates and gas).

3. Briefly about the angular degrees of freedom, utilized in the three-dimensional elements PRADIS

It is known that solid body of one angular position into another can be transferred by one turning around a certain axis, called axis of final rotation (Euler's theorem). Let us designate e1, e2, e 3- the direction cosines of axis of final rotation, Fi - angle of final rotation. Then it is possible to introduce four kinematic parameters, that describe the angular motion of solid body [1,2]:

x1 = e1 * sin (Fi/2),
x2 = e2 * sin (Fi/2), (1)
x3 = e3 * sin (Fi/2),
x4 = cos (Fi/2),

and one equation of relation for these parameters:

x1 * x1 + x2 * x2 + x3 * x3 + x4 * x4 = 1 (2)

In contrast to any set of three kinematic parameters (in particular - the Euler angles) the indicated four parameters they do not degenerate with any position of solid body, (i.e. go to infinity neither parameters themselves nor speed of their change).

The angular degrees of freedom, accepted in the three-dimensional elements PRADIS, are expressed as kinematic parameters (1) as follows:

q1 = x1 * Lq,
q2 = x2 * Lq, (e)
q3 = x3 * Lq,
q4 = x4 * Lq,

where

Lq = sqrt (q1 * q1+q2 * of q2+q3 * of q3+q4 * q4). (4)

The first three degrees of freedom are external for the models of elements, the fourth - internal, hidden before the user. The initial value of the potential variable, corresponding internal degree of freedom, is set in the models of elements equalequal to 1.

Flow variables for the first three degrees of freedom are moments along the global axes of the X, Y, Z. the fourth (internal) flow variable it holds in control change in the time of value Lq (4):

i4 = Mu * d (Lq)/dT, (shch)

where Mu - constant of proportionality, identical for all degrees of freedom of this type and taken in the models of elements to the equal

Mu = DABSI/of sqrt (MSHEPS). (')

What operations, from the point of view of user, are correct with the work with three external angular degrees of freedom of the models of elements? Almost all methods, characteristic for forward motion, remain valid and in this case.

In particular:

- it is possible to forbid (basing the appropriate units) motion according to the selected angular degrees of freedom, which is equivalent to the reduction of the dimensionality of the vector, directed along the axis of final rotation (for example, with two fixed angular degrees of freedom, point it can revolve only around the axis, which corresponds to the loose unit);

- connection in the direction of the rotation between the points of abutting members it is also possible to achieve (if this is necessary) not according to all three degrees of freedom, but only on those selected.

With which it is necessary to be more careful? In contrast to the flat rotation, the first and second derivatives of the potential variables (e) will not be angular velocity and angular acceleration respectively. Therefore the, for example, initial conditions, given by model VN, will not, in the general case, determine initial angular velocity. It is natural that also [PRVP] of type the V and A will derive not angular velocity, but instantaneous values of the first and second derivative of the potential variable. However, the values of angular velocities and accelerations are accessible from the working vector of some models of elements, in particular J3O.

4. Literature:

1. Vittenburg I. Dynamics of the systems of solid bodies. - M.: MIR, 1980.