Peculiarities of technical objects' mathematical models generation by means of PRADIS program system

This paper describes the calculation kernel peculiarities of PRADIS program system intended to analyse the transient processes in time domain. We considered here the main theses and peculiarities of the approach, applied for generating the library of dimensional mechanical elements within the limits of created program system. We also give some examples of practical usage of the developed software in the analysis of automotive modules' dynamics.

PRADIS program system is intended to analyse transient processes in technical systems in time domain. Under it's realization developers were guided by methodology adopted in schematic design packages (1) and, first of all, in «PA-6» complex (2). Two main advantages of the latter – universality regarding problem domain of analysis and openness for expanding the elements models library – made it attractive for creating the applications in different problem domains (3).

What was the reason for development of the new program system called PRADIS? The main reason – problem of software portability. «PA-6» complex, implemented in assembler language for EC computer (like IBM 360), became inaccessible when another type of computer facilities was installed. When number of computer and operating system types constantly increases, the portability factor is a very important criterion for effective service life of the software.

PRADIS software was developed for application in problem domain, that's why during PRADIS development it was necessary to take into account the specific character of problem domain. The main application – analysis of equipment assemblies and vehicles, i.e. mechanical and mixed mechanical (hydro-, pneumo-, electromechanical) systems. It was important to take into account this specific character either during work on algorithm of invariant part of the system or during development of elements models library. This article is devoted to these peculiarities.

Peculiarities of calculation kernel

Object's mathematical model (MM) generation and analysis methods, used by PRADIS, are traditional for program systems of similar purposes. They are:

- nodal method for MM generation;
- implicit methods for numerical integration of differential equations (DE) set;
- Newton method for iterative solution of the nonlinear algebraic and transcendental equations set;
- Gaussian method for linear algebraic equations set solution.

Below we shortly describe the following peculiarities of calculation kernel's working algorythms:

- Jacobian generation functions split between integration program and elements' models;
- algorithms for minimization of array structure filling-up

- Linear algebraic equations set decision program implementation peculiarities. Nodal method of forming MM in it's classical form operates in every node of one variable of potential type, analogue of which in mechanics is speed, in hydraulics - pressure. The set of first-order differential equations is generated according to this approach. To solve this set we use methods oriented namely on first-order DE. As to dynamics of mechanical systems, the main relation here is the equation of motion, which is second-order differential equation. From practical point of view this peculiarity becomes apparent on the level of elements' models, which are responsible for calculation of data-flow variables (forces and torques in mechanics) and local Jacobians. Traditional approach presupposes the transmission of current values of potential variables into element model using the following law - one variable describes the status of one node. As a result, model of any hardness-inertial element, which is the most frequently found element in mechanics, must make preliminary current values calculations for displacements and accelerations in element nodes (degrees of freedom) on the basis of transmitted current speeds. Calculation algorithm must be coordinated in some way with used integration method formulas, i.e. definite binding of elements models to used integration method type appears.

That's why in PRADIS we implement the nodal method modification, which is characterised by the following peculiarities:

- generation of second-order differential equations set of the following form

$$F(u, \frac{du}{dt}, \frac{d^2u}{dt^2}, t) = 0 \qquad (1)$$

where **u** – vector of potential variables (displacement in mechanics);

- To solve system (1) the implicit one-step integration methods are used (Shtermer's method, Newmark's method), based on differential formulas of the following form

$$\begin{aligned} u_i &= f_1(u_{i-1}, du_{i-1}/dt, d^2u_i/dt^2, \Delta t_i) \\ du_i/dt &= f_2(du_{i-1}/dt, d^2u_i/dt^2, \Delta t_i) \end{aligned} \qquad (2)$$

- From the elements models point of view, variable and its derivatives are mutually independent potential variables; partial flow derivatives are given by each newly generated potential variables:

$$\begin{aligned} &Y_u = \partial F/\partial u;\ Y_{u'} = \partial F/\partial u'; \\ &Y_{u''} = \partial F/\partial u''. \end{aligned} \qquad (3)$$

These values are transformed into Jacobian matrix using dependencies (2) by the integration program. Algebraic linear equations set available for every Newton method interaction is solved with Gaussian method.

Parent matrix rows renumber for the purpose of minimazing secondary filling are obtained by one of three methods:

- Rows increasing of the nonzero parent matrix elements ordering (I);
- Using small-scale algorithm (the last from the rest rows having minimum active elements number from the start of the exception step can be taken as dominating row) (II);
- Using small-scale local fill algorithm (the last from the rest rows at this exception step provide minimum second fill) (III).

Summarazing these renumber methods practical experience for solving the different problems a comparison was made in terms of speed and effectiveness (optimum matrix structure).

1. The application of method I is expedient for the sets with the number of equations < 100. In this case second filling has no time to distortion the prime pattern and the results remain acceptable. For sets containing more than 200 equations second filling growth becomes catastrophic;
2. The algorithms I and III enter a competition for sets consist of 100-5000 equations. Upper level is relative and may be vary with the problem. For the same sets algorithm II time on the average slightly less and the number of the second nonzero elements as a rule slightly higher than for algorithm III. On the whole for such equations sets method results remain about the same;
3. For the sets containing from 10 000 and more equations method II gives less satisfactory results. At least it is true for the tasks obtaining during discretization of sampling topological rectangle equivalent domains. The number of second nonzero elements dramatically increases as well as algorithm time. Method III provides the linear growth of the memory and decision time, demanded for solving linear algebraic equations, depending on the task size at least for sets containing 20 000 – 25 000 equations. During rare linear algebraic equation solving program realization by Gaussian method emerges the necessity for arrange search cycles inside the internal cycles of the downward process of elimination. Its considerably slows down the program.

In PA-6 is used approach that after optimum numbering presupposes using linear algebraic equation decision program generation without iterative and conditional statements useful only for solving one specific problem. PRADIS authors abandoned the linear algebraic equation decision program generation because of excessively high demands to memory and compliers limitation on the code dimension. At the expense of some algorithm reorganization we managed to avoid internal branching and additional search cycle. At the moment two internal decision cycles of linear algebraic equation used in PRADIS looked as follows:

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DO 200 J = BEGEL, MIDEL

C  regular leading line of the next block – CNR (J).
     NUMEX = CNR (J)
     FIRST = HA (2, NUMEX) + 1
     LAST = HA (3, NUMEX)
     T      = - A (J+N) / A (NUMEX)
     B (I)  = B (I) + T * B (NUMEX)
C  elimination cycle

     DO 100  K = FIRST, LAST
     A (VECTOR (CNR (K))) = A (VECTOR (CNR (K))) + T * A (K+N)

 100     CONTINUE
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Advantages PARADIS elements models library

The advantage of schematic design approach for PA-6, PA-7 is an opportunity of fast receiving mathematical models of the complex engineering systems. However in models of this type model parameters may not correspond to initial design values of the subject of investigation. In PA-6 developers and package users attempted to create elements allowing to operate with design values on obtaining engineering system models [3].

This line of investigation was explored further in the PA-7. PRADIS library of elements accounts for experience of the previous developments. Currently it is based on elements with parameters defined in terms of a product being designed (geometrical dimensions, mass-inertia characteristics and rheological characteristics of the material). As we often have to tackle problems of a mixed physical nature, elements which allow taking into account mutual influence of the processes occurring in the different subsystems, occupy a prominent place in the library of elements [4], [5].

One of the PRADIS outstanding features is an availability of the developed library of 3D elements. Let us consider some questions specific to three-dimensional motion. In all, movement of a free solid body may be separated into progressive displacement with reference to some pole and rotational motion about this pole. Three of six independent coordinates defining solid body motion specify the pole progressive displacement and, consequently, another three ones determine its rotation. In the case under study we are interested in rotational component of the motion, because the same basic relationships of 2D elements remain true for progressive degrees of freedom. When considering rotation of a body we need first of all to choose kinematic parameters corresponding to angular degrees of freedom. The severity of the problem is that in case of 3D rotation integral of the angular velocity taken as some finite interval of time in no way determines body angular position comparing to the progressive displacement (or to 2D rotation) where velocity and acceleration are the direct derivatives of displacement. Thus, in general case body finite angular velocity cannot be uniquely determined knowing its initial position and three scalars resulting from integration of three projections of angular velocity onto fixed coordinate axes over some interval of time.

A number of different systems of kinematic parameters is known which are used for definition of the solid rotational motion: direction cosines, Euler and Krylov angles, Cayley-Klein parameters, Rodrig-Hamilton parameters [6]. It would be reasonable to use a system with minimum number of kinematic parameters, i.e. 3, that is equal to the number of independent degrees of freedom to determine rotational motion. For example, widely known Euler angles meet this requirement. However, any such system involving a set of three kinematic parameters is negatively characterized by the following: at certain angle values kinematic equations are degenerated when either parameter by itself or its derivative become indeterminate.

Rogrig- Hamilton parameters (which in some papers are referred to as “Euler parameters”) based on the well known Euler theorem have not this essential drawback. According to the Euler theorem, a solid body can be moved from one angular position to another by a single rotation about some axis called “finite rotation axis”. We introduce notation: n_1, n_2, n_3 are the direction cosines of the finite rotation axis, • is the finite rotation angle. Then we can determine 4 kinematic parameters specifying solid body angular motion

$$\begin{cases} q_i = n_i \sin(\frac{\varphi}{2}), i = 1,3 \\ q_4 = \cos(\frac{\varphi}{2}) \end{cases} \tag{4}$$

and one equation of connection:

$$(\sum q_i^2 = 1), i = 1,4 \tag{5}$$

From mathematical point of view, these kinematic parameters represent normalized quaternions.

Potential variables corresponding to rotational degrees of freedom in 3D mechanical elements of PRADIS system are represented by kinematic parameters (4) in the following manner:

$$x_i = q_i L_q, i = 1,4 \tag{6}$$

where

$$L_q = \sqrt{\sum x_i^2}, i = 1,4 \tag{7}$$

Flow variables for the first three degrees of freedom are moments on the X, Y, Z global axes. The fourth flow variable holds back L_q change with respect to time:

$$I_4 = \mu \frac{dL_q}{dt} \tag{8}$$

where μ is a constant of proportionality common to all degrees of freedom of this kind.

When developing models of elements, the following relations are additionally used [6]:

- relationships for coordinates transformation matrix which allow to determine the current angular orientation of the mobile basis;
- kinematic equations expressing angular velocity vector components in terms of the values of kinematic parameters and the derivatives.

A designer of deformable element models may also need relations for calculation of angular deformation components between two points at the known values of kinematic parameters in them. In this case a rule of the finite turns should be used implying that the components of the resulting turn quaternion are determined from the first and the second turn components according to quaternion multiplication rule.

Let us consider some issues of practical use of the element models having spatial motion rotational degrees of freedom.

From a mechanic-user point of view to work with four degrees of freedom of rotational motion somewhat unnatural. Therefore, only first three degrees of freedom with that the flow variables have clear physical meaning – the moments influencing the element – are taken as external degrees of freedom available for the user at the stage of the model structure description. The fourth degree of freedom is made internal, concealed from the user. In such a form the diagram of any element having spatial motion rotational degrees of freedom looks natural (Fig.1).

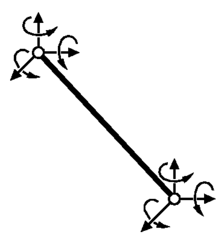

Figure 1. An elastic beam is an example of a 3-D element having rotational degrees of freedom.

What are correct operations while working with three external angular degrees of freedom of element models (from user's point of view)? Practically all the techniques characteristic of progressive motion remain true in this case, in particular:

- The sources of force action can be directly connected to spatial angular degrees of freedom and in this case they will reproduce torque action around the relevant coordinate axis;
- The motion in the chosen angular degrees of freedom can be forbidden (basing the relevant nodes) that is tantamount to vector length reduction directed along the axis of finite rotation (e.g. Having two fixed angular degrees of freedom the point may rotate only around the axis corresponding to the unfixed node);
- Rotational coupling between the points of the butted elements can also be executed (if it is necessary) only in the chosen degrees of freedom, not in all three ones.

The operations of direct kinematic action with the aid of sources of kinematic actions meant for progressive motion and two-dimensional rotation will be incorrect. These sources specify the law of variation of a potential variable associated with displacement, speed or acceleration. But in this case, potential variables basis on the spatial angular degrees of freedom represents a set of kinematic parameters and their time derivatives, which are not angular speed or angular acceleration. Therefore, in case it is required to transfer the rotation from one- or two-dimensional elements to spatial ones it is necessary to use coupling elements such as a part of the shaft having at one end a rotational degree of freedom around the shaft's axis, and at the other one – three rotational degrees of freedom around coordinate axes (Fig.2).

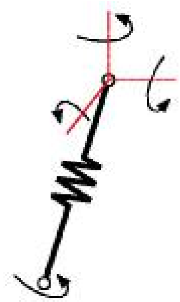

Figure 2. An elastic coupling element for rotational motion transition from one-dimensional to spatial elements.

3-D mechanisms analysis using the elements based on the described approach can be illustrated by an example taken from car engine computational data.

Figure 3 shows the configuration of the mechanism consisting of a crankshaft, a connecting rod and a piston gear, supports, a flywheel, outer impacts in the form of cyclically changing forces induced by a combustion chamber, resistance to a camshaft rotation and a belt gear-produced load of cantilever-type. In the task the radial runouts in the base bearing and the loading of hazard-prone areas of the crankshaft at the engine speed up to 5 000 rev/min were studied. The crankshaft areas were represented as the beam springing elements showing the crankshaft hardness (properties) and inertia elements including mass distribution and moments of inertia. To obtain the crankshaft hardness and inertia parameters, its detailed finite element model was constructed. The beam inertia model characteristics were specified in conformity with its behavior.

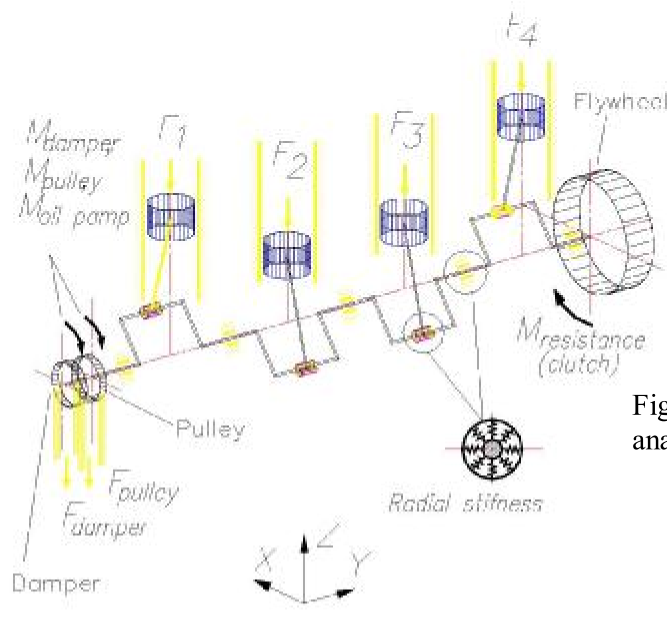

Figure 3a. Crankshaft acceleration analysis in the car engine:
The mechanism and external loads scheme.

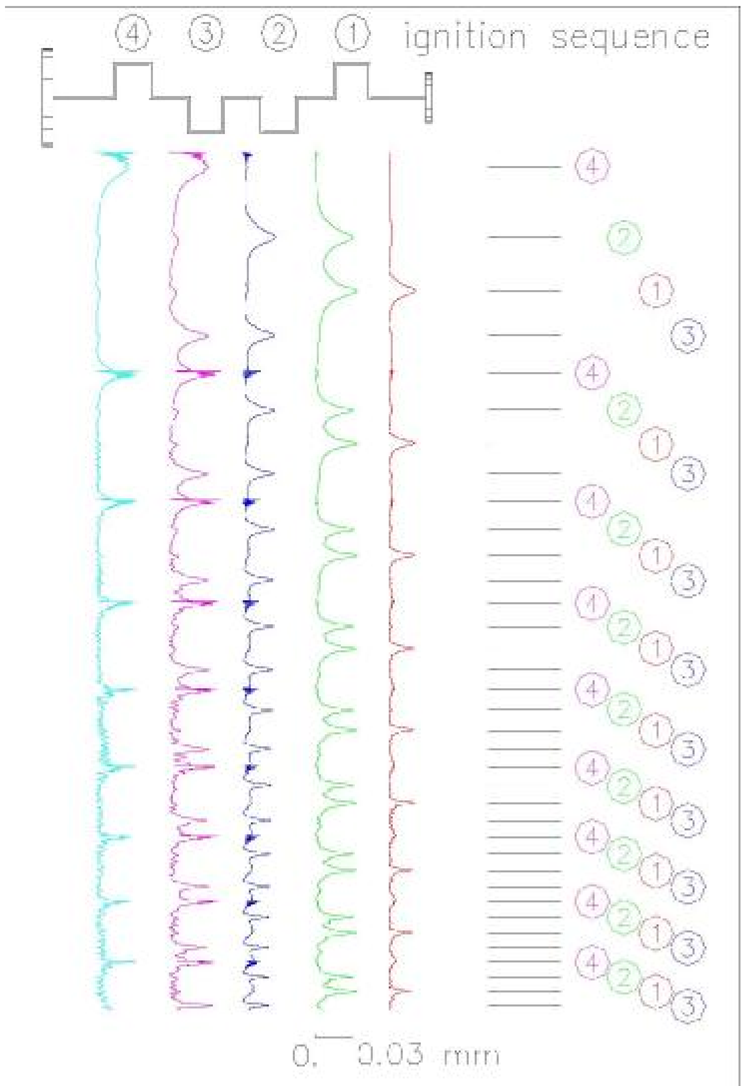

Figure 3b. Crankshaft acceleration analysis in the car engine
Calculation results – radial runouts in the base bearing.

It should be noted that three rotational degrees of freedom are not necessarily be provided for every point of some 3D elements of the models developed. For instance, lateral offset representation in the form of full polynomial is desirable for modeling 3D triangle plate element. The full polynomial consists of either six terms (quadratic) or ten ones (cubic). Construction of ten term polynomial for the triangle element that might perform unspecified displacements, requires minimal 22 degrees of freedom (7 degrees of freedom for every point: three progressive, four rotational including three outer and one inner degrees; and one offset in the centre of the element). The element modeled using quadratic polynomial is, firstly, more cost-effective due to only 12 degrees of freedom are available (three progressive degrees in every point and one rotational degree in the middle of each side (Fig. 4). Secondly, this kind of element features better angular deformation continuity at its bound crossing that provides more accurate solutions in many cases [7]. The distinctive feature of this plate is that its angular degrees of freedom are the degrees of freedom of rotation about the

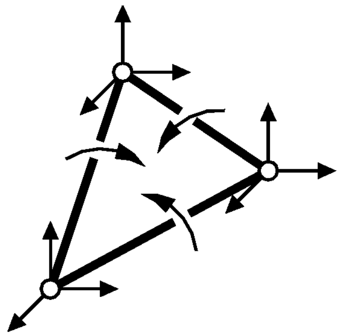

Fig. 4. Triangle 3D plate with rotational degrees of freedom about its sides.

relative sides, but not about the coordinate axes. Due to this, the plates of this kind are mainly intended for the finite element areas description when the angular degrees of freedom are connected via the homogeneous elements. Figure 5 shows the example of the task that was solved using the elements of this type.
The impact testing procedure of a steering wheel was modeled using a dummy head that was impacted into the zone where the rim and hub are connected. The dummy head of a globe form had 165 cm in diameter and 6,8 kg in weight. Its velocity prior the impact amounted to 7 m/sec. The highest and longest registered values (above 3 msec) were estimated in accordance with the safety requirements to the steering wheel.

The main features of the PRADIS program system that are concerning the mathematical modeling of production equipment include the following:

- mathematical modeling in the form of the 2d order DE (differential equation) and the use of implicit numerical methods of integration that are the 2d order DE-oriented;
- computation of Jacobian elements in the model separately from the displacements, velocities and accelerations (or their analogues for other potential variables) supporting the model elements independence of the integration methods;
- adaptive choice (in respect to the task dimensionality) of optimal node renumbering at the step of factorization;
- object model construction using mainly structure element models that allows to receive input data directly from the design documentation;
- availability of extended library of mechanical and mixed mechanical elements that allows to reduce the different-type object development time;
- modeling of mechanical elements with 3D rotational degrees of freedom using potential variables in the form of nonnormalized quaternions excludes degeneration of kinematic parameters at any angular position of the element and minimizes the number of additional equations of connection;
- computation efficiency increase in case of analyzing tasks of high dimensionality that are complicated for calculation (contact, plasticity) due to use of elements with special angular degrees of freedom about the movable axes.

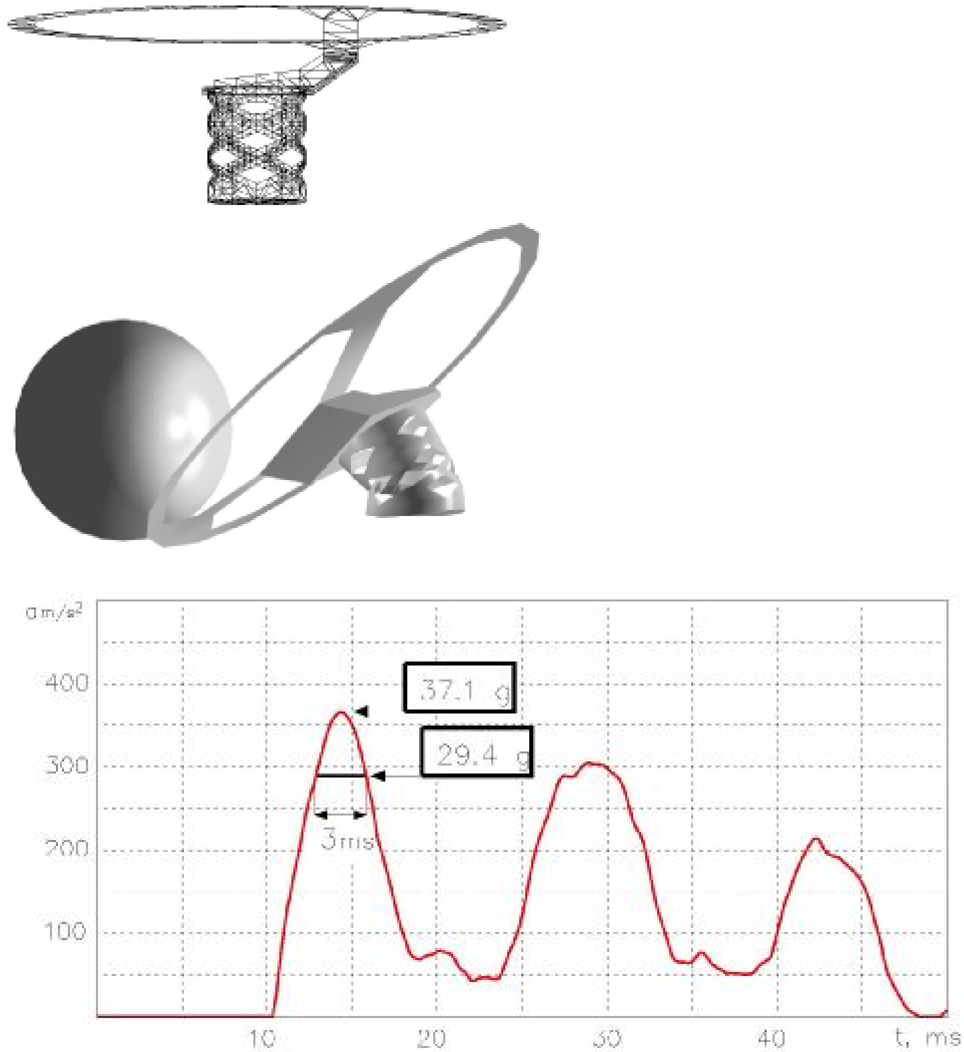

Fig.5. The VAZ 2108 steering wheel impact test modeling:

a) finite element model of the wheel;
b) deformation computations pattern;
c) dummy head deceleration during the impact.